Stability of flux compactifications and the pattern of supersymmetry breaking

K. Choi [a], **A. Falkowski** [b], **H. P. Nilles** [c], **M. Olechowski** [b], **S. Pokorski** [b]

[a] *Department of Physics, Korea Advanced Institute of Science and Technology*
Daejeon 305-701, Korea

[b] *Institute of Theoretical Physics, Warsaw University*
ul. Hoża 69, PL-00 681 Warsaw, Poland

[c] *Physikalisches Institut, Universität Bonn*
Nussallee 12, D-53115 Bonn, Germany

Abstract

We extend the KKLT [1] approach to moduli stabilization by including the dilaton and the complex structure moduli into the effective supergravity theory. Decoupling of the dilaton is neither always possible nor necessary for the existence of stable minima with zero (or positive) cosmological constant. The pattern of supersymmetry breaking can be much richer than in the decoupling scenario of KKLT.

1 Introduction

One of the central questions in superstring theory is the stabilization of moduli. In general, the moduli fields include the dilaton S, the Kähler moduli T_i and the complex structure moduli Z_i. Early attempts in the heterotic string theory tried to fix the dilaton S with a combination of gaugino condensation and nontrivial flux of the 3-form field strength H [2]. The stabilization of S turned out to be inherently connected to supersymmetry breakdown. More recently it was observed [3], that in the framework of Type II B theory one could fix complex structure moduli as well as the dilaton with a combination of 3-form fluxes of the field strengths F_3 and H_3 [4], even in the absence of supersymmetry breakdown. These attempts were only partially successful, as the moduli directions T_i remained flat. Without a stabilization of the remaining moduli, it is difficult to draw specific conclusions about properties of the theories such as soft supersymmetry breaking terms.

In a more recent paper [1] (KKLT) a combination of 3-form flux (in the Type II B theory) and gaugino condensation was argued to lead to complete stabilization of all moduli. The analysis in KKLT is done in the framework of a low energy supergravity approximation. They assume that the dilaton and the complex structure moduli (CSM), if present, have been fixed by H_3 and F_3 fluxes and concentrate on an effective theory for the volume modulus T (representing the Kähler moduli T_i). This decoupling procedure is self-consistent if the masses for S and CSM are much larger than the mass of T. Non-perturbative corrections to the superpotential are then used to stabilize T. In this analysis the ground state of the theory had a large negative vacuum energy (AdS space) and preserved supersymmetry. To reach an acceptable potential KKLT proposed a so-called uplifting mechanism that breaks supersymmetry (in a local minimum) and allows a fine tuning of the cosmological constant to a desired value (e.g. de Sitter space).

The purpose of the present paper is twofold. First, we include into the effective supergravity theory the dilaton and the CSM.[1] We compare in a rather model independent way the results of such an extended analysis with the KKLT results obtained under the decoupling assumption. Secondly, we carefully analyze the pattern of supersymmetry breaking in our extended framework. We shall see that a meaningful statement about the soft supersymmetry breaking terms is only possible once all the moduli are stabilized.

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[1]A similar extension of the KKLT scenario has been considered in ref. [5].

The soft terms show an unexpectedly rich structure already in the (decoupled) KKLT limit and even more so in the general set-up.

We adopt the same assumptions for the supergravity analysis as in KKLT. In particular, we assume that the expectation values of T and S are large. In the region of large S and T, there are simple formulae for the defining functions of supergravity: the Kähler potential K, the gauge kinetic function f and the superpotential W. We concentrate on two specific schemes: $(D3/7)$, the original KKLT scenario in type II B theory with matter on $D3$ [1] and/or $D7$ branes [6] and (H), a heterotic theory with flux stabilization on a non-Calabi-Yau manifold [7] and gaugino condensation [8].

Our results can be summarized as follows (all within the approximation stated above):

- In the first step (flux compactification without gaugino condensation) there are stable minima but there remains an unstabilized modulus (the flat direction T in the Type II B case or the runaway direction S in the heterotic case),

- After inclusion of gaugino condensation the existence of local (supersymmetric) minima in the large S, T region is not guaranteed in all cases. It depends strongly on the model under consideration. In models without complex structure moduli, there do not exist stable supersymmetric AdS minima in the large S, T region. We find, with a quite general uplifting potential that for D3/7 a stable minimum does not exist also after uplifting. Thus, in D3/7 models without CSM the KKLT procedure of decoupling S is inconsistent. For the heterotic model, a stable minimum does exist after uplifting.

- In models with CSM, we derive the conditions on the (effective) superpotential that allow for the existence of the supersymmetric AdS vacuum, with all moduli stabilized. These conditions are restrictive but more general than the decoupling limit of KKLT. Therefore, the KKLT mechanism of stabilization works for a more general class of models than those in which both CSM and S can be decoupled. The same conditions are sufficient for the existence of a stable local minimum after uplifting.

- For the effective superpotential satisfying the conditions allowing for a stable minima, we discuss the supersymmetry breaking parameters F_S

and F_T. We find that several other options than the KKLT limit $F_S \ll F_T \ll m_{3/2}$ are possible, with potentially interesting consequences for phenomenology.

We stress here again that these results are obtained within the supergravity approximation and the assumptions concerning K, f and W as given explicitly in section 2, where we set up our notation and review the mechanism of KKLT. Section 3 is devoted to an analysis of the simplest case, with just the dilaton S, one Kähler modulus T and no complex structure moduli Z_i. In section 4 we include complex structure moduli in a rather model independent way. Section 5 contains an analysis of supersymmetry breaking soft terms in the uplifted minima and the consequences for realistic model building. Conclusions and outlook follow in section 6. In the present paper we shall try to avoid the presentation of detailed technicalities of the calculations. These will be relegated to a future publication where also more examples are presented.

2 Supergravity description of flux compactifications

2.1 Notation and conventions

We concentrate on the case with the dilaton S, a Kähler modulus T and complex structure moduli Z_i. Matter superfields are denoted by Q. We assume to be in a region of large S and T. Let us start with the D3/7-system [9, 10, 11]. The Kähler potential is assumed to be

$$K = -\log(S + \overline{S} - |Q_7|^2) - 3\log(T + \overline{T} - |Q_3|^2) + \tilde{K}(Z_i, \overline{Z}_i) \qquad (1)$$

where Q_3, Q_7 denote matter multiplets on the D3, D7 branes, respectively. The gauge kinetic functions are

$$f_3 = S \qquad \text{or} \qquad f_7 = T \qquad (2)$$

for gauge bosons on the D3 or D7 branes. The superpotential is given by

$$W = W(S, Z_i) + C \exp(-aT) + W(Q_3, Q_7), \qquad (3)$$

where C and a are constants. The term $C \exp(-aT)$ represents nonperturbative effects as explained in KKLT. When analyzing the potential we look

for minima where Q_7 and Q_3 scalars (and therefore $W(Q_i)$ as well) do not receive nontrivial vacuum expectation values. These minima coincide in the two cases $D3$ and $D7$, while the consequences for the soft supersymmetry breaking terms might differ.

In the heterotic case (H) we have

$$
\begin{aligned}
K_H &= -\log(S + \overline{S}) - 3\log(T + \overline{T} - |Q_H|^2) + K(Z_i, \overline{Z}_i) \\
f_H &= S \\
W &= W(T, Z_i) + C\exp(-aS) + W(Q_H),
\end{aligned}
\tag{4}
$$

where $C\exp(-aS)$ comes from gaugino condensation in the hidden sector and $W(T, Z_i)$ comes from nontrivial fluxes on specific non-Calabi-Yau manifolds [7]. As an example we consider here the compactification on half-flat manifolds as given in ref. [12].

2.2 The proposal of KKLT

The authors of ref. [1] consider the D3/7 case and assume that the dilaton and complex structure moduli are heavy compared to T, such that they can be integrated out leaving a constant contribution W_0 to the effective low-energy superpotential $W_{\text{eff}} = W_0 + C\exp(-aT)$. This leads to a potential with a supersymmetric AdS minimum in which the (last) modulus T is fixed as well. To this end, they add an uplifting potential $\Delta V(T, \overline{T})$ that breaks supersymmetry and allows a local minimum at a (fine tuned) small positive vacuum energy. The same proposal can be realized in the heterotic case where T, Z_i are integrated out and S is fixed by gaugino condensation.

One might now be interested in the analysis of the mechanism of supersymmetry breakdown and the properties of the soft terms. The sources of supersymmetry breakdown are vacuum expectation values (vevs) of the auxiliary components of the moduli superfields: F_S, F_T and F_{Z_i}, and also the auxiliary component of the 4D supergravity multiplet. In the KKLT scheme, however, all but one of the moduli have been integrated out. Therefore meaningful statements can only be made for F_T in the D3/7 case (or F_S in the heterotic case). The information on the remaining moduli is hidden in W_0. For a full analysis of the nature of supersymmetry breakdown we would have to go a step back to a more fundamental level and "integrate in" the other moduli. Only then can we make meaningful statements about the relation of the values of the various auxiliary fields and check if the assumption about integrating out S was consistent.

3   The two modulus case: S and T

The simplest setup is offered by models without complex structure moduli (CSM) Z_i. Alternatively one might consider models where the complex structure moduli have been integrated out. Later, however, we shall see that this mechanism of integrating out moduli might be problematic and therefore we here explicitly assume the absence of CSM. We shall treat the two cases separately.

3.1   Type IIB case: D3/7

We consider the D3/7 system with two moduli $T = t + i\tau$ and $S = s + i\sigma$ with the Kähler potential and superpotential given by:

$$K = -3\log(T + \overline{T}) - \log(S + \overline{S}) \,,$$
$$W = A + BS + Ce^{-aT} \tag{5}$$

where A, B, C and a are constants. The exponent a is real and positive. For simplicity we choose parameters A, B and C to be real (the results in the general case are unchanged). We choose B and C in such a way that $B \cdot C < 0$. Then we may restrict our analysis to the stationary point with vanishing axion vevs $\tau = \sigma = 0$ (for positive $B \cdot C$ we can just shift τ by π/a).

   The supersymmetric stationary point of the potential can be found by solving the equations $F_S = F_T = 0$, where $F_X \equiv -e^{K/2}(K'')^{-1Y}_X D_Y W$, $D_Y W \equiv (\frac{\partial K}{\partial Y} W + \frac{\partial W}{\partial Y})$. It occurs for the values of s and t satisfying the constraints:

$$\frac{Ce^{-at}}{A} = -\frac{3}{at + 3} \,, \qquad \frac{Bs}{A} = \frac{at}{at + 3} \,. \tag{6}$$

These constraints can be solved for s and t with the help of the Lambert W function but the above form is more useful for our analysis. In the following we will assume that the parameters of the superpotential are such that the supersymmetric stationary point at large positive values of at and s (of order 10 or larger) does exist.

   In order to determine the nature of the stationary points we have to consider the second derivatives of the potential V_0 at those points. The explicit calculation shows, that all mixed real-imaginary derivatives vanish $\frac{\partial^2 V_0}{\partial t \partial \tau} = \frac{\partial^2 V_0}{\partial t \partial \sigma} = \frac{\partial^2 V_0}{\partial s \partial \tau} = \frac{\partial^2 V_0}{\partial s \partial \sigma} = 0$ and the second derivative matrix splits into

two 2×2 matrices, V''_{0M} and V''_{0A}. After inserting the relations (6) we find that all diagonal entries in these matrices are positive. Nevertheless both the moduli and the axionic mass matrix has one positive and one negative eigenvalue. The instability can be easily seen by calculating the determinant of these matrices

$$\det V''_{0M} = -\frac{3B^4}{32M_p^4 s^4 t^8}(4a^2 t^2 + 13at + 10),$$

$$\det V''_{0A} = -\frac{3B^4}{32M_p^4 s^4 t^8}at(4at + 3) \tag{7}$$

which are negative for all $at > 0$. We conclude that the supersymmetric point is a saddle point with instabilities along the moduli and axionic directions. One can also check that other, non-supersymmetric extrema of V_0 are also saddle points. Furthermore, there exist directions in the (t, τ, σ) field space for which $\lim_{s \to 0^+} V_0 = -\infty$ and the potential is unbounded from below. This limit is, of course, outside the validity of the perturbative supergravity approximation. Still we can conclude that within the region of large S and large T there are no local minima. All the stationary points are saddle points.

This shows that in this case the mechanism of KKLT faces problems. The difference can be explained in the following way. KKLT first considered the case with $C = 0$ and a fixed dilaton. Then they integrated out the dilaton before they included the gaugino condensate (assuming implicitly that $m_s, m_\sigma \gg m_t, m_\tau$). The mass of the dilaton, however, still depends on the T-modulus and therefore this procedure is not necessarily justified. Our analysis keeps S as well as T after including the gaugino condensate and shows that the assumption of KKLT is not justified at this level, as the masses of S and T are comparable.

In principle, of course, one might say that even in the absence of (supersymmetric) AdS minima one might arrive at a stable situation after uplifting. We study a quite general lifting potential of the form:

$$\Delta V = \frac{D}{(T + \bar{T})^{n_t}(S + \bar{S})^{n_s}} \tag{8}$$

and consider the potential $V = V_0 + \Delta V$. We assume that the exponents n_t and n_s are positive or zero integers. We look for solutions of the equations $\frac{\partial V}{\partial T} = \frac{\partial V}{\partial S} = 0$ with the parameter D fine-tuned such that $V = 0$ (or a small positive value). Such analysis shows that the situation in the D3/7 cases

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remains unstable. We do not find any stable uplifted vacua in the limit of large at. Both moduli and axionic instabilities persist, independently of the exponents in the lifting potential. In some cases there exist stable vacua at the small value of the moduli, $at \sim 1$, but in this region the perturbative approximation of the potential might not be reliable.

3.2 Heterotic case: H

We perform the similar analysis in the heterotic case. Here we take

$$K = -3\log(T + \overline{T}) - \log(S + \overline{S}),$$
$$W = A + BT + Ce^{-aS}. \tag{9}$$

The supersymmetric stationary points exist if the parameters satisfy the constraints: $C \cdot A > 0$, $B \cdot A < 0$ and $|C| \gg |A|$. They occur for the values of s and t satisfying the conditions:

$$\frac{Ce^{-as}}{A} = \frac{1}{as - 1}, \qquad \frac{Bt}{A} = -\frac{3as}{as - 1}. \tag{10}$$

Again the second derivative matrix at the stationary point splits into two 2×2 blocks with all diagonal entries positive. This time the moduli matrix V_M'' has two positive eigenvalues for $as > 2$. But the instability is present in the axionic sector - the determinant of V_A'' is negative for $as > 0$. We conclude that the supersymmetric point is a saddle point with an instability along the axionic direction. One can also check that other, non-supersymmetric extrema of V_0 are also saddle points. Therefore for the heterotic system as well we do not obtain local minima in the region of validity of our approximation. The behavior of the limiting values of the potential is similar to that in the D3/7 case. There are directions in the moduli space for which the potential is unbounded from below for $s \to 0^+$.

Note, however, that the situation is somewhat different than for the D3/7 systems which have both moduli and axionic instabilities at the supersymmetric point. Moreover, the character of the axionic mass matrix can be changed by a small perturbation of the vacuum solution in eq. (10). Whether the stationary point is a minimum or a saddle point depends on the determinant of this matrix, which vanishes in the leading order in $1/(as)$. The unstable character of the supersymmetric stationary point is determined only by the subleading terms.

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As we have seen, the supersymmetric stationary points in the heterotic case are only marginally unstable. Thus small corrections to the Kähler potential might lead to stable minima, while this is not possible in the D3/7 case. In fact it can be shown that any correction to K making $(\partial_T K)^2/\partial_T^2 K > 3$ can lead to a stable supersymmetric AdS minimum. In this regard, an interesting possibility is to have the higher order sigma model correction yielding $K = -\ln\left[(T + \bar{T})^3 + E\right]$ where E is a positive constant depending on the topological data of the underlying compact manifold [13]. We have analyzed this case as well as other modifications of the Kähler potential which will be presented in a future publication.

The lifting potential can also provide for a small correction to the vacuum solution and, under certain conditions, the stationary points could become stable minima. We add the lifting potential of eq. (8) and look for vacuum solutions in the large as limit and with vanishing cosmological constant. For $n_t \neq 1$ the solution for which $C\exp(-at)/A$ behaves as $1/(as)$ (similarly as the supersymmetric solution) reads:

$$\frac{Ce^{-as}}{A} = \frac{(n_t - 4)}{4(n_t - 1)}\frac{1}{as} + \mathcal{O}\left(\frac{1}{(as)^2}\right), \qquad \frac{Bt}{A} = -\frac{3(n_t - 2)}{2(n_t - 1)} + \mathcal{O}\left(\frac{1}{as}\right). \quad (11)$$

Only for $n_t = 0$ the above solution coincides with the supersymmetric solution (10) in the large as limit. For $n_t > 1$ the lifted solution is shifted with respect to the supersymmetric solution already at the leading order in $1/(as)$. The moduli directions stay stable for $n_t < 1$. For $n_t > 1$ the moduli potential develops a saddle point instability (the case $n_t = 1$ is more complicated and will be analyzed fully elsewhere). Calculating the determinant of the axionic mass matrix we find that for $n_t = 0$ the axionic direction is also stable as long as $n_s \geq 1$.

4 Inclusion of the complex structure moduli

4.1 Type IIB case: D3/7

We now include CSM Z_i in the analysis. That is we study the system described by

$$K = -3\log(T + \overline{T}) - \log(S + \overline{S}) + \tilde{K}(Z_i, \bar{Z}_i),$$
$$W = W_{\text{flux}} + Ce^{-aT} = A(Z_i) + B(Z_i)S + Ce^{-aT}. \quad (12)$$

In this case $W_{ij} = \frac{\partial^2 W_{\text{flux}}}{\partial Z_i \partial Z_j}$ and $W_{iS} = \frac{\partial^2 W_{\text{flux}}}{\partial Z_i \partial S}$ at supersymmetric vacuum are non-vanishing in general [14]. If $W_{ij} > W_{iS}$, Z_i can be integrated out first, leaving an effective superpotential for S. In the following, we consider such situation that Z_i are heavy enough to be integrated out, while m_S has an arbitrary value between m_{Z_i} and $m_{3/2}$.

The resulting effective superpotential $W_{\text{eff}}^S[S]$ depends on the precise form of $A(Z_i)$ and $B(Z_i)$. We parametrize our ignorance by assuming that integrating out Z_i leaves some general function of S in the effective superpotential. Of course one might have assumed that the case discussed in the previous section, with $W_{\text{eff}}^S = A + BS$, has been obtained after integrating out CSM. We have already learned that the conclusions of KKLT do not hold in this specific case. We will study the equations of motion for general $W_{\text{eff}}^S[S]$ and determine the conditions that W_{eff}^S must satisfy in order to arrive at a stable vacuum. In the next section we also study the dependence of soft breaking terms on W_{eff}^S. Thus, we consider the supergravity setup defined by:

$$K = -3\log(T + \overline{T}) - \log(S + \overline{S}),$$
$$W = W_{\text{eff}}^S[S] + Ce^{-aT}. \tag{13}$$

Again, for simplicity we assume that C and $W_{\text{eff}}[s]$ are real. The supersymmetric stationary points occur for s and t satisfying the constraints:

$$\frac{Ce^{-at}}{W_{\text{eff}}^S} = -\frac{3}{3 + 2at}, \qquad \frac{sW_{\text{eff}}^{S\,'}}{W_{\text{eff}}^S} = \frac{at}{3 + 2at}. \tag{14}$$

To determine the character of the stationary point we must study the second derivatives. The relevant parameter here is γ defined as:

$$\gamma \equiv \frac{s\, W_{\text{eff}}^{S\,''}}{W_{\text{eff}}^{S\,'}} \tag{15}$$

with s satisfying the constraints (14). The second derivative matrices after inserting the relation (14) read:

$$V_{0M}'' = \frac{W_{\text{eff}}^{S\,'2}}{8M_p^2 st^5} \begin{bmatrix} 3s^2(4a^2t^2 + 10at + 7) & 3st(2at + 3 - 2\gamma) \\ 3st(2at + 3 - 2\gamma) & t^2(1 - 2\gamma + 4\gamma^2) \end{bmatrix} \tag{16}$$

for the moduli (t, s) and

$$V_{0A}'' = \frac{W_{\text{eff}}^{S\,'2}}{8M_p^2 st^5} \begin{bmatrix} 3s^2(4a^2t^2 + 6at + 3) & 3st(2at + 1 - 2\gamma) \\ 3st(2at + 1 - 2\gamma) & t^2(1 + 2\gamma + 4\gamma^2) \end{bmatrix} \tag{17}$$

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for the axions (τ, σ). All diagonal entries are manifestly positive. Whether a stationary point is a minimum or a saddle point depends on the signs of the determinants. The general conditions for positivity of the determinants are rather complicated. The case $\gamma = 0$ was studied in the previous section and we found instabilities in both axionic and moduli direction. Therefore the KKLT setup with a simple linear superpotential $W_{\text{eff}}^S[S] = A + BS$ is inconsistent. Stability can be achieved only when the second derivative of the S-dependent effective superpotential at the minimum $s^2 W_{\text{eff}}^{S\,\prime\prime}$ is at least comparable to W_{eff}^S and $sW_{\text{eff}}^{S\,\prime}$, that is for γ of order one or bigger. In particular, in the large $(a\,t)$ limit the stability condition is very simple:

$$|\gamma| > 1 \tag{18}$$

For $\gamma \gg 1$ and $\gamma \gg at$ the S modulus becomes much heavier than T and is decoupled. This is the KKLT limit. For $\gamma \to \infty$ we should recover all the results of KKLT. The actual magnitude of γ depends on the flux compactification model under consideration. Note that, by vacuum equations (14), large at of order 10 implies $C \gg sW_{\text{eff}}^{S\,\prime}$, thus $W_{\text{eff}}^{S\,\prime}$ must be suppressed with respect to the string scale, such that $sW_{\text{eff}}^{S\,\prime}/C < 10^{-3}$. This can be achieved if the flux parameters are appropriately fine-tuned or if there exist some hierarchy of parameters in W_{eff}^S. In the former case one would expect $\gamma \gg 1$ (unless another fine-tuning makes $W_{\text{eff}}^{S\,\prime\prime}$ suppressed as well), in the latter case $\gamma \sim 1$ can be natural.

In order to arrive at a vacuum with a vanishing (or small positive) cosmological constant we include the lifting potential eq. (8). The analysis is quite complicated and we shall restrict ourselves to consider only the large $(a\,t)$ limit (which includes the KKLT limit when $\gamma \gg at$). In the former case, after eliminating D and t we find the vev of s is given by the solution of the equation

$$(n_s - 1)W_{\text{eff}}^{S\,2} + 2s(n_s + 1)W_{\text{eff}}^S W_{\text{eff}}^{S\,\prime} - 2n_s s^2 W_{\text{eff}}^{S\,\prime 2} + 2s^2 W_{\text{eff}}^S W_{\text{eff}}^{S\,\prime\prime} - 4s^3 W_{\text{eff}}^{S\,\prime} W_{\text{eff}}^{S\,\prime\prime} = 0 \tag{19}$$

This can be easily solved for W_{eff}^S to give the vacuum solution in a form similar to eq. (14). The most compact form is obtained for $n_s = 1$ and in the following we restrict to this case (for $n_s \neq 1$ there is no qualitative difference). In this special case we get:

$$\frac{Ce^{-at}}{W_{\text{eff}}^S} = -\frac{3}{2at} + \mathcal{O}\left(\frac{1}{(at)^2}\right), \qquad \frac{sW_{\text{eff}}^{S\,\prime}}{W_{\text{eff}}^S} = \frac{2+\gamma}{1+2\gamma} + \mathcal{O}\left(\frac{1}{at}\right). \tag{20}$$

The stability condition is somewhat more complicated (it depends also on $W_{\text{eff}}^{S}{}'''$) but generically it requires $\gamma \gtrsim 1$ as in the AdS$_4$ vacuum. We see that, in general, even in the large (at) limit the lifted vacuum is shifted with respect to the position of the supersymmetric AdS$_4$ vacuum, see eq. (14). Only for $\gamma \gg 1$ the position of the lifted vacuum coincides with that of the supersymmetric vacuum (in fact eqs. (20) hold for arbitrary n_s then). The lifted vacuum is always stable in this limit.

4.2   Heterotic case: H

One can do the analogous analysis for the heterotic case defined by

$$
K = -3\log(T + \overline{T}) - \log(S + \overline{S}) \,,
$$
$$
W = W_{\text{eff}}^{T}[T] + Ce^{-aS} \tag{21}
$$

where $W_{\text{eff}}^{T}[T]$ represents the T-dependent part of the superpotential after integrating out the CSM. The supersymmetry preserving configurations satisfy the following conditions for the moduli t and s:

$$
\frac{Ce^{-as}}{W_{\text{eff}}^{T}} = -\frac{1}{1 + 2as}\,, \qquad \frac{t\,W_{\text{eff}}^{T}{}'}{W_{\text{eff}}^{T}} = \frac{3as}{1 + 2as}\,. \tag{22}
$$

The character of those stationary points depends on the second derivative matrices which are given by

$$
V''_{0M} = \frac{W_{\text{eff}}^{T}{}'^{2}}{72M_p^2 s^3 t^3}\begin{bmatrix} 3s^2(4\eta^2 - 10\eta + 7) & 3st(2as + 3 - 2\eta) \\ 3st(2as + 3 - 2\eta) & t^2(4a^2s^2 + 2as + 1) \end{bmatrix}\,, \tag{23}
$$

$$
V''_{0A} = \frac{W_{\text{eff}}^{T}{}'^{2}}{72M_p^2 s^3 t^3}\begin{bmatrix} 3s^2(4\eta^2 - 6\eta + 3) & 3st(2as + 1 - 2\eta) \\ 3st(2as + 1 - 2\eta) & t^2(4a^2s^2 - 2as + 1) \end{bmatrix} \tag{24}
$$

where t and s satisfy conditions (22) and the parameter η is defined by

$$
\eta \equiv \frac{tW_{\text{eff}}^{T}{}''}{W_{\text{eff}}^{T}{}'}\,. \tag{25}
$$

After calculating determinants of the above matrices we find that in the leading order in $1/(as)$ the supersymmetric stationary points (22) are stable minima for

$$
|\eta - 1| > 1\,. \tag{26}
$$

One can see that in the absence of CSM the heterotic model with $\eta = 0$ is just at the border of the (in)stability region.

The supersymmetric stationary points (stable or not) satisfying eq. (22) have negative vacuum energy. We can lift them to zero or small positive value by adding the lifting potential of the form (8). The conditions for the minima with vanishing energy are in general quite complicated. In the leading order in $1/(as)$ the values of the moduli s and t at such minima (for $n_t \neq 3$) satisfy:

$$\frac{Ce^{-as}}{W_{\text{eff}}^T} = -\frac{1}{2as} + \mathcal{O}\left(\frac{1}{(as)^2}\right) , \qquad \frac{tW_{\text{eff}}^{T\,\prime}}{W_{\text{eff}}^T} = \frac{3(2 - n_t - \eta)}{4 - n_t - 2\eta} + \mathcal{O}\left(\frac{1}{as}\right) . \quad (27)$$

These conditions coincide in leading order with the conditions before lifting given by eq. (22) only for $n_t = 0$ or in the large η limit.

5 Soft supersymmetry breaking terms

In this section, we discuss the soft SUSY breaking terms in various setups we have considered so far. We will concentrate on soft terms induced by the auxiliary components of S, T and the 4D supergravity multiplet under the assumption that CSM are heavy enough to be ignored. We also ignore the soft terms that might possibly be induced by the direct couplings between the visible sector and the uplifting sector.

Before presenting the results for the vevs of the auxilary components in KKLT compactifactions let us summarize the important facts about supersymmetry breaking mediation in supergravity [15, 16, 17]. The auxiliary component of 4D supergravity multiplet can be parameterized by a chiral compensator $\phi = 1 + \theta^2 \hat{F}_\phi$ whose F-component is given by

$$\hat{F}_\phi = m_{3/2} + \frac{1}{3} F_A \partial_A K = m_{3/2} - \frac{1}{3}\hat{F}_S - \hat{F}_T , \quad (28)$$

where $\hat{F}_A = F_A/(A + \bar{A})$. The auxiliary component $\hat{F}_\phi$ can induce soft terms (other than B) only at loop-level by the mechanism of anomaly mediation [16]. As we will see, in some interesting limits of flux compactifications, $\hat{F}_\phi$ is bigger than $\hat{F}_{S,T}$ by one or two orders of magnitudes, e.g. $\hat{F}_\phi = \mathcal{O}\left(8\pi^2 \hat{F}_{S,T}\right)$. In this case, the loop-induced soft terms associated with $\hat{F}_\phi$ can be equally important as the dilaton and/or modulus-mediated soft terms at tree-level.

To accomodate the loop-induced soft terms, let us consider the superspace lagrangian of the visible fields which includes quantum corrections at the compactification scale M_c:

$$
\begin{aligned}
\mathcal{L}_{\text{visible}} &= \int d^4\theta \left[Y_i \bar{Q}_i Q_i + \frac{1}{16} \left(G_a W^{a\alpha} \frac{D^2}{\partial^2} W^a_\alpha + \text{h.c} \right) \right] \\
&+ \left[\int d^2\theta \frac{1}{6} \lambda_{ijk} Q_i Q_j Q_k + \text{h.c} \right],
\end{aligned} \tag{29}
$$

where the holomorphic Yukawa couplings λ_{ijk} are assumed to be moduli-independent constants. At tree level, G_a corresponds to the holomorphic gauge kinetic function f_a. In 4D supergravity, the superspace lagrangian of a Kähler potential K is given by $\int d^4\theta[-3e^{-K/3}]$, thus

$$
Y_i = (S + \bar{S})^{1/3} (T + \bar{T}) \mathcal{Z}_i,
$$

for $K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T}) + \mathcal{Z}_i \bar{Q}_i Q_i$. From (1), (2) and (4), one easily finds the tree-level expressions of G_a and Y_i for the D3,D7 and heterotic matter/gauge fields:

$$
\begin{aligned}
G_3^{(0)} = G_H^{(0)} = S, \qquad G_7^{(0)} = T, \\
Y_3^{(0)} = Y_H^{(0)} = (S + \bar{S})^{1/3}, \qquad Y_7^{(0)} = (S + \bar{S})^{-2/3}(T + \bar{T}),
\end{aligned} \tag{30}
$$

where the superscript (0) means the tree-level result.

The physical gauge and Yukawa couplings renormalized at M_c are given by

$$
\frac{1}{g_a^2} = \text{Re}(G_a), \quad y_{ijk} = \frac{\lambda_{ijk}}{\sqrt{Y_i Y_j Y_k}}. \tag{31}
$$

Let us define the canonically normalized gaugino masses, A-parameters and scalar masses as

$$
\frac{1}{2} M_a \lambda^a \lambda_a - \frac{1}{2} m_i^2 \left| \tilde{Q}_i \right|^2 - \frac{1}{6} A_{ijk} y_{ijk} \tilde{Q}_i \tilde{Q}_j \tilde{Q}_k + \text{h.c.} \tag{32}
$$

Generically these soft masses are given by

$$
\begin{aligned}
M_a &= c_a^S \hat{F}_S + c_a^T \hat{F}_T + c_a^\phi \hat{F}_\phi, \\
A_{ijk} &= a_{ijk}^S \hat{F}_S + a_{ijk}^T \hat{F}_T + a_{ijk}^\phi \hat{F}_\phi, \\
m_i^2 &= h_i^{S\bar{S}} \left| \hat{F}_S \right|^2 + h_i^{T\bar{T}} \left| \hat{F}_T \right|^2 + \left(h^{S\bar{T}} \hat{F}_S \bar{\hat{F}}_T + \text{h.c} \right) \\
&+ h_i^{\phi\bar{\phi}} \left| \hat{F}_\phi \right|^2 + \left(h_i^{S\bar{\phi}} \hat{F}_S \bar{\hat{F}}_\phi + h_i^{T\bar{\phi}} \hat{F}_T \bar{\hat{F}}_\phi + \text{h.c} \right).
\end{aligned} \tag{33}
$$

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In our case including the limit with $\hat{F}_\phi = \mathcal{O}(8\pi^2 \hat{F}_{S,T})$, the dominant parts of soft masses at M_c can be determined by the tree-level values of c_a^A, a_{ijk}^A and $h_i^{A\bar{B}}$ $(A, B = S, T)$, the one-loop values of c_a^ϕ, a_{ijk}^ϕ and $h^{A\bar{\phi}}$, and also the two-loop values of $h_i^{\phi\bar{\phi}}$. Inserting (30) into the superspace lagrangian (29), we find the following tree-level results for the D3,D7 and heterotic matter/gauge fields [15]:

$$c_3^S = c_H^S = 1, \quad c_3^T = c_H^T = 0, \quad c_7^S = 0, \quad c_7^T = 1,$$
$$a_{ijk}^A = \kappa_i^A + \kappa_j^A + \kappa_k^A, \quad h_i^{A\bar{A}} = \kappa_i^A, \quad h_i^{A\bar{B}} = 0 \ (A \neq B), \qquad (34)$$

where

$$\kappa_3^S = \kappa_H^S = \frac{1}{3}, \quad \kappa_3^T = \kappa_H^T = 0, \quad \kappa_7^S = -\frac{2}{3}, \quad \kappa_7^T = 1.$$

The one-loop values of c_a^ϕ and a_{ijk}^ϕ and the two-loop values of $h_i^{\phi\bar{\phi}}$ correspond to the well-known anomaly-mediated soft terms [16]: c_a^ϕ corresponds to the one-loop beta function coefficient $\frac{dg_a}{d\ln\mu} = c_a^\phi g_a^3$, while $a_{ijk}^\phi = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_j)$ and $h_i^{\phi\bar{\phi}} = -\frac{1}{4}\frac{d\gamma_i}{d\ln\mu}$ where $\gamma_i = \frac{d\ln Y_i}{d\ln\mu}$ is the anomalous dimension of Q_i. These anomaly-mediated soft masses can be most easily computed by including the ϕ-dependent one-loop corrections of G_a and Y_i in the superspace lagrangian (29) [16, 17]:

$$\mathrm{Re}(G_a) = \mathrm{Re}(f_a) - \frac{1}{16\pi^2}\left(3T_a(\mathrm{Adj}) - \sum_i T_a(Q_i)\right)\ln\left(\frac{\phi\bar{\phi}}{\mu^2}\right) + ..., \qquad (35)$$

$$\ln(Y_i) = \ln\left(Y_i^{(0)}\right) - \frac{1}{32\pi^2}\left(4\sum_a \frac{T_a(Q_i)}{\mathrm{Re}(f_a)} - \sum_{jk}\frac{|\lambda_{ijk}|^2}{Y_i^{(0)}Y_j^{(0)}Y_k^{(0)}}\right)\ln\left(\frac{\phi\bar{\phi}}{\mu^2}\right) + ...,$$

where T_a denotes the quadratic Casimir and the ellipses stand for the ϕ-independent (but generically moduli-dependent) loop corrections which are not relevant for us. These ϕ-dependent parts of G_a and Y_i determine $h^{A\bar{\phi}}$ also as

$$\Delta m_i^2 \equiv h^{S\bar{\phi}}\hat{F}_S\bar{\hat{F}}_\phi + h^{T\bar{\phi}}\hat{F}_T\bar{\hat{F}}_\phi + \mathrm{h.c}$$
$$= \frac{1}{32\pi^2}\left(\sum_{jk}|y_{ijk}|^2 A_{ijk}^{(0)} - 4\sum_a g_a^2 T_a(Q_i) M_a^{(0)}\right)\bar{\hat{F}}_\phi + \mathrm{h.c}, \quad (36)$$

where $M_a^{(0)} = c_a^S \hat{F}_S + c_a^T \hat{F}_T$ and $A_{ijk}^{(0)} = a_{ijk}^S \hat{F}_S + a_{ijk}^T \hat{F}_T$ are the tree-level gaugino masses and A-parameters.

Let us now discuss the relative ratios between $\hat{F}_S$, $\hat{F}_T$ and $\hat{F}_\phi$ that we found in KKLT compactifications. For the D3/7 system in the limit $at \gg 1$ with $n_s = 1$, we find

$$m_{3/2} \approx \frac{sW_{\text{eff}}^{S\,'}}{2M_p^2} \frac{1+2\gamma}{4+2\gamma} s^{-1/2} t^{-3/2} \,,$$

$$\hat{F}_S \approx -\frac{3}{1+2\gamma} m_{3/2} \,,$$

$$\hat{F}_T \approx \frac{1}{at} \frac{6 - n_t + \gamma(3+2n_t) + 2\gamma^2 n_t}{(1+2\gamma)^2} m_{3/2} \,. \tag{37}$$

From (20), one easily finds $at \sim \ln(M_p/m_{3/2})$. If $m_{3/2}$ is of the order of the TeV scale then at can be as large as 35. In such case, $1/at$ is comparable to the one-loop suppression factor, $at = \mathcal{O}(8\pi^2)$. The above results show that $\hat{F}_S = \mathcal{O}(m_{3/2})$ and $\hat{F}_T = \mathcal{O}(m_{3/2}/at)$ for $\gamma \sim 1$. In this case, all soft terms on D3 brane and also the scalar masses on D7 brane are dominated by the contributions from $\hat{F}_S$, while the gaugino masses and A-parameters on D7 branes receive equally important contributions from $\hat{F}_T$ and $\hat{F}_\phi$.

On the other hand, in the KKLT limit of the D3/7 system in which $m_S \gg m_T$ and thus $|\gamma| \gg at$, one always finds (for arbitrary n_s) $\hat{F}_T = \mathcal{O}(m_{3/2}/at)$ and $|\hat{F}_S| \ll |\hat{F}_T|$. Thus in the KKLT limit with weak scale supersymmetry in which $at \sim \ln(M_p/m_{3/2}) = \mathcal{O}(8\pi^2)$, soft terms on D3 brane are dominated by anomaly-mediation, while those on D7 brane are dominated by the equally important contributions from $\hat{F}_T$ and $\hat{F}_\phi$.

In the heterotic case without CSM, we were also able to find a stable vacuum solution after including the lifting potential. Here the pattern of soft breaking terms is different. For $n_t = 0$ and $as \gg 1$, we find

$$m_{3/2} \approx \frac{A}{2M_p^2} s^{-1/2} t^{-3/2} \,,$$

$$\hat{F}_S \approx -\frac{1}{as} \frac{3n_s}{2} m_{3/2} \,,$$

$$\hat{F}_T \approx \frac{1}{as} \frac{3n_s}{8} m_{3/2} \,. \tag{38}$$

Again (11) indicates $as \sim \ln(M_p/m_{3/2})$. Although $\hat{F}_S \sim \hat{F}_T$, the soft masses from $\hat{F}_T$ appear only at higher orders in either string loop expansion or α'-expansion due to the no-scale nature. Thus, in this heterotic setup with weak

scale supersymmetry, soft masses are dominated by the equally important contributions from $\hat{F}_S$ and $\hat{F}_\phi$.

For the heterotic case with CSM and the lifting potential with $n_s = 1$, supersymmetry breaking parameters for $as \sim \ln(M_p/m_{3/2}) \gg 1$ are given by

$$m_{3/2} \approx \frac{tW_{\text{eff}}^{T\,\prime}}{12M_p^2} \frac{4 - n_t - 2\eta}{2 - n_t - \eta} s^{-1/2}t^{-3/2} \,,$$

$$\hat{F}_S \approx \frac{1}{as} \frac{3(2 - n_t - \eta)}{4 - n_t - 2\eta} m_{3/2} \,,$$

$$\hat{F}_T \approx \frac{n_t}{4 - n_t - 2\eta} m_{3/2} \,. \tag{39}$$

For $\eta \sim 1$ and $n_t \neq 0$, we have $\hat{F}_T = \mathcal{O}(m_{3/2})$, while $\hat{F}_S = \mathcal{O}(m_{3/2}/as)$. In this case, all of $\hat{F}_S$, $\hat{F}_T$ and $\hat{F}_\phi$ give similar contributions to soft masses. On the other hand, in the KKLT limit of heterotic set up in which $m_T \gg m_S$ and thus $|\eta| \gg 1$, soft terms are dominated by the equally important contributions from $\hat{F}_S$ and $\hat{F}_\phi$. This feature is very similar to the D3/7 case if we interchange s with t and γ with η.

6   Conclusions and outlook

We have extended the KKLT approach to moduli stabilization by explicitly including the dilaton (or T in the heterotic model) and the complex structure moduli into the effective supergravity theory. The conditions on the effective superpotential for the dilaton (or T) field have then been derived that allow for the existence of supersymmetric AdS vacua and a stable minimum after supersymmetry breaking and uplifting of the scalar potential. These conditions provide restrictions on the CSM and the flux configurations once explicit string models are constructed. However, they admit a much more general class of models than those in which the dilaton S (Kähler modulus T in the heterotic case) can be decoupled. In consequence, the pattern of supersymmetry breaking as given in equations (37-39) can be much richer than $F_S \ll F_T \ll m_{3/2}$ (as in the KKLT limit where anomaly mediation plays an important role) and there are regions of parameter space in which tree-level mediation becomes dominant. Therefore a generalization of the scheme beyond the KKLT limit seems to be required before definite conclusions can be drawn. A more complete discussion of this situation together with the full technical details will be the subject of a future publication [18].

Acknowledgments

Note added

While completing this work we became aware of a paper by Kallosh and Linde [19] where the simplest KKLT version has been modified. Also this version is formulated in the KKLT limit ($|\gamma| \gg at$) and should be generalized along the lines discussed above before the full picture of the pattern of supersymmetry breakdown can be analyzed.

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